The Tango of Rotating Black Holes and Spinning Particles

YU-TIN HUANG
NATIONAL TAIWAN UNIVERSITY

The no-hair theorem states that black holes are characterized by only their mass, charge and angular momentum. As these are the same quantum numbers that are associated with elementary particles, a natural question arises: to which extent does a black hole mimic an elementary particle? A naive answer would be that at sufficiently large distances, the dynamics of black holes (e.g., the orbital motion of in-spiral black holes) should be well approximated by that of an elementary particle. For example, in the limit where the impact factor $b \ll p, m$, where (m, p) are the mass and spatial momenta of the black holes, we expect that the dynamics of binary black holes to be well described by the gravitational interactions of elementary particles. On the face of it, this proposal does not seem particularly enlightening, for at large distances, doesn't everything look like a point particle?

Let us take a closer look at how point particles behave in a gravitational background. A covariant description of point particles in a non-trivial background is the worldline formalism, where the degrees of freedom are the worldline fields $x^\mu(\tau)$ and spin-operators $S_{\mu\nu}$ that encodes the spin degrees of freedom. A general worldline action in non-trivial gravitational background takes the form [1, 2, 3]:

$$S = \int d\tau \left\{ -m\sqrt{u^2} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + L_{SI}\left[u^\mu, S_{\mu\nu}, g_{\mu\nu}x^\mu(\tau)\right] \right\} \tag{1}$$

where, $u^\mu \equiv \dfrac{dx^\mu}{d\tau}$ and $\Omega_{\mu\nu}$ is the angular velocity. The first two terms of the Lagrangian are universal irrespective of the details of the point-like particle, while the terms in L_{SI} correspond to spin-induced multipole terms that are beyond minimal coupling and depend on the inner structure of the particle.

They can be parameterized as:

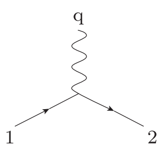
$$\begin{aligned} L_{SI} = &\sum_{n=1}^{\infty} C^{2n} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} \\ &+ \sum_{n=1}^{\infty} C^{2n+1} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n}} S^{\mu_{2n+1}} \end{aligned} \tag{2}$$

where E and B are the electric and magnetic components of the Riemann tensor, and S^μ are the spin vectors extracted from the spin operators via $S^{\mu\nu} = -\frac{1}{m}\epsilon^{\mu\nu\rho\sigma} p_\rho S_\sigma$. The coefficients $\{C^{2n}, C^{2n+1}\}$ are Wilson coefficients that describe how a spinning particle interacts with the gravitational background, which requires detailed knowledge of the spinning particle at hand. In other words, not only are spinning particles in general distinct, their differences can be characterized though an infinite number of unknown parameters! For a spinning black hole, the complete Wilson coefficients were only derived recently [4].

From the discussion above we see that by considering a spinning black hole, the spin multipoles introduce an infinite number of "probes" that allow us to characterize the nature of black holes even at long distances. Now, let's try to connect black holes with elementary particles.

While the black holes and elementary particles share the same set of quantum numbers, for black holes the spins are really classical, as compared to the quantized spins for elementary particles. Thus in making the connection, what we really have in mind is an analytic extrapolation of spin $S \to \infty$ with $\hbar \to 0$ and $S\hbar$ held fixed. On the face of it, this appears to be a daunting task, since we need to characterize the couplings of spinning particles with gravitational fields in such a way that an extrapolation can be made. At an even deeper level, the proposal doesn't make sense physically since we know that elementary particles, in the sense that they are devoid of internal structure, do not exist beyond spin-2.

The above difficulty was circumvented by considering on-shell matrix elements: instead of starting with fields, Lagrangians and then deriving physical observables, we can work with the observables directly, which often times are completely determined from the kinematics and symmetry alone. A few years ago, N. Arkani-Hamed, T. C. Huang and the author introduced a novel "spinor-helicity formalism" to parameterize general scattering matrix elements in four-dimensions [5]. This formalism utilizes kinematic variables that are free of any constraint and transform covariantly under the Little group, which is the symmetry group under which physical states form representations. The simplest three-point amplitude for an electromagnetically and gravitationally coupled spinning particle is given by:



$$= g(xm)^h \frac{\langle \mathbf{12} \rangle^{2S}}{m^{2S}} \tag{3}$$

where h = (1,2) and, $g = \left(\sqrt{2}e, \frac{\kappa}{2}\right)$ for positive photons and gravitons respectively. Importantly, the variable x defined kinematically as:

$$q \cdot p_1 = 0 \to x\lambda_\alpha = \frac{(p_1)_{\alpha\dot{\alpha}}\,\tilde{\lambda}^{\dot{\alpha}}}{m} \tag{4}$$

where we've used the bi-spinor representation of momenta $(p)_{\alpha\dot{\alpha}} = p^\mu(\sigma_\mu)_{\alpha\dot{\alpha}}$ and since $q^2 = 0$, $q_{\alpha\dot{\alpha}} = \lambda_\alpha\tilde{\lambda}_{\dot{\alpha}}$.

When Eq. (3) was first written down, for S = $\frac{1}{2}$, 1 it was quickly matched with QED and electroweak photon couplings, as well as gravitationally minimally coupled spinors and vectors. An immediate question was what kind of higher spin particles can match to Eq. (3) for general S? In other words, what kind of higher spin physical state couples to electromagnetic and gravitational fields minimally? It turns out, in the S $\to \infty$ limit we have a definite answer, it is a rotating black hole! Indeed, in subsequent work [8, 7], by taking the classical spin limit of Eq. (3), it was shown that Wilson coefficients for the multipole moments of a Kerr black hole were correctly reproduced. Note that the simplicity of black hole couplings in such a kinematic basis can also be viewed as the on-shell avatar of the no-hair theorem.

This simplicity has led to the streamlined computation of spin effects in the scattering angle [8], linear and angular impulse [9] of rotating black holes, giving the complete 1 PM (post-Minkowskian) conservative Hamiltonian of spinning binary black holes [10], as well as new insights into the origin of shift relations between rotating and Schwarzschild black hole solutions [11].

The identification of black hole dynamics with a minimally coupled spinning particle leads to several conceptual challenges. We know that at the end of the day, elementary particles cannot exceed spin-2. On the scattering amplitude side, this constraint shows up at the four-point amplitude, where there is no longer a well-defined notion, even kinematically, of minimal coupling. However, the very same amplitude enters into the determination of the spin-dependent terms in the 2 PM Hamiltonian, which should be uniquely determined from general relativity (GR). In other words, there appears to be a tension between the scattering of elementary particles and the uniqueness of classical black holes. What is the principle that ultimately resolves this? Furthermore, while we have identified minimal coupling kinematically, by simply requiring that in some suitable basis, the coupling contains minimal momentum dependence. Is there a more physical property that can be utilized to characterize such a coupling? The fact that for spin $-\frac{1}{2}$ minimal coupling appears to be favored for entanglement maximization, is a tantalizing possibility to explore [12].

References

[1] W. D. Goldberger and I. Z. Rothstein, "An Effective field theory of gravity for extended objects," Phys. Rev. D 73, 104029 (2006) [hepth/0409156].

[2] R. A. Porto, "Post-Newtonian corrections to the motion of spinning bodies in NRGR," Phys. Rev. D 73, 104031 (2006) [gr-qc/0511061].

[3] M. Levi and J. Steinhoff, "Spinning gravitating objects in the effective field theory in the post-Newtonian scheme," JHEP 1509, 219 (2015) [arXiv:1501.04956 [gr-qc]].

[4] J. Vines, "Scattering of two spinning black holes in post-Minkowskian gravity, to all orders in spin, and effective-one-body mappings," Class. Quant. Grav. 35, no. 8, 084002 (2018) doi:10.1088/1361-6382/aaa3a8 [arXiv:1709.06016 [gr-qc]].

[5] N. Arkani-Hamed, T. C. Huang and Y. t. Huang, "Scattering Amplitudes For All Masses and Spins," arXiv:1709.04891 [hep-th].

[6] A. Guevara, A. Ochirov and J. Vines, "Scattering of Spinning Black Holes from Exponentiated Soft Factors," JHEP 1909, 056 (2019) doi:10.1007/JHEP09(2019)056 [arXiv:1812.06895 [hep-th]].

[7] M. Z. Chung, Y. T. Huang, J. W. Kim and S. Lee, "The simplest massive S-matrix: from minimal coupling to Black Holes," JHEP 1904, 156 (2019) doi:10.1007/JHEP04(2019)156 [arXiv:1812.08752 [hep-th]].

[8] A. Guevara, A. Ochirov and J. Vines, "Scattering of Spinning Black Holes from Exponentiated Soft Factors," JHEP 1909 (2019) 056 [arXiv:1812.06895 [hep-th]].

[9] A. Guevara, A. Ochirov and J. Vines, "Black-Hole Scattering with General Spin Directions from Minimal-Coupling Amplitudes," Phys. Rev. D 100 (2019) no.10, 104024 [arXiv:1906.10071 [hep-th]].

[10] M. Z. Chung, Y. t. Huang, J. W. Kim and S. Lee, "Complete Hamiltonian for spinning binary systems at first post-Minkowskian order," arXiv:2003.06600 [hep-th].

[11] N. Arkani-Hamed, Y. t. Huang and D. O'Connell, \Kerr black holes as elementary particles," JHEP 2001, 046 (2020) doi:10.1007/JHEP01(2020)046 [arXiv:1906.10100 [hep-th]].

[12] A. Cervera-Lierta, J. I. Latorre, J. Rojo and L. Rottoli, \Maximal Entanglement in High Energy Physics," SciPost Phys. 3, no. 5, 036 (2017) doi:10.21468/SciPostPhys.3.5.036 [arXiv:1703.02989 [hep-th]].

Yu-tin Huang received his PhD degree in 2009 from YITP at SUNY Stony Brook University. He went on to post doctoral positions at UCLA and University of Michigan, and became a member at Institute of Advance Studies in Princeton in 2013. In 2014 he was appointed assistant professor at NTU. He is one of the leading experts in Scattering amplitudes, and received the 2018 Nishiha Asia award as well as 2018 Ta-You Wu memorial award and is the Golden-Jade fellow of CTP/NTU.