Award of Nobel Prize to Roger Penrose - an Appreciation of His Contributions to General Relativity

James Vickers
Mathematical Sciences and STAG Research Centre, University of Southampton, University Road, Southampton, SO17 1BJ, UK

[Reproduced from the IUPAP NewsLetter]

This year's Nobel Prize in Physics was awarded to Roger Penrose "for the discovery that black hole formation is a robust prediction of the general theory of relativity" and Reinhard Genzel and Andrea Ghez "for the discovery of a supermassive compact object at the centre of our galaxy". The press release goes on to refer to Penrose's 1965 paper *Gravitational Collapse and Space-Time Singularities* as "still regarded as the most important contribution to the general theory of relativity since Einstein". Indeed despite known examples of singular solutions to the field equations Einstein rejected their physical reality writing that "singularities of the field are to be excluded". Indeed, until the 1960s it was the prevailing opinion of most researchers that the known singular solutions were an artefact of the high degree of symmetry or were unphysical is some way. For this reason the Penrose singularity theorem has been described as the first genuine post-Einsteinian result in general relativity.

As noted above it was widely believed that singularities were not generic features of general relativity. This perception was backed up by the work of Lifshitz and Kalatnikov who used a function counting argument which suggested that singular solutions contained fewer arbitrary functions than the general solution to the field equations and could therefore be regarded as of measure zero. All this changed when in 1965 Penrose published the first of the modern singularity theorems which considered gravitational collapse without assuming any symmetry assumptions. Despite its short length, of only 3 pages, the paper introduced many of the concepts that continue to play a key role in our analysis and understanding of the structure of spacetime. Most importantly he introduced the fundamental notion of a trapped surface. He then showed that if a spacetime possesses both a closed trapped surface and a non-compact Cauchy surface then as long as the local energy density is always positive, so that via Einstein's equations the Ricci tensor satisfies the null convergence condition, the spacetime cannot be future null complete. The paper thus established the idea of using geodesic incompleteness to characterise a singular spacetime and showed for the first time that the gravitational singularity found in the Schwarzschild solution was not a result of the high degree of symmetry. So as long as the gravitational collapse qualitatively resembles the spherically symmetric case, in the sense that a closed trapped surface is formed, then deviations from spherical symmetry cannot prevent the formation of a gravitational singularity. The 1965 paper had immediate impact and inspired a series of papers by Hawking, Penrose, Ellis, Geroch and others which led to the development of the modern singularity theorems. Of particular note are the Adam's prize essay of Penrose "An analysis of the structure of space-time", the corresponding essay by Hawking entitled "Singularities and the geometry of space-time" and the 1970 Hawking-Penrose singularity theorem which in the words of the abstract "implies that space-time singularities are to be expected if either the universe is spatially closed or there is an 'object' undergoing relativistic gravitational collapse (existence of a trapped surface) or there is a point whose p past null cone encounters sufficient matter that the divergence of the null rays through p changes sign somewhere to the past". The impact of the 1965 paper was not just in the result that singularities were a stable feature of space-time but just as importantly in the new methods used to establish the result as demonstrated by the title of his 1972 monograph "Techniques of Differential Topology in General Relativity". In particular the notion of closed trapped surface has had an enormous influence and continues to play a key role not only in understanding the physics of black holes, but also in numerical relativity and cosmology.

Since Einstein's equations break down at a singularity it is not clear how the physics involved with the formation of a singularity will causally influence the future. In the case of the Schwarzschild solution the singularity is hidden from the exterior region so this is not an issue outside the black hole. In his 1969 paper on gravitational collapse (which also introduced the "Penrose process" for extracting energy from a rotating black hole) Penrose suggested that this might be the general situation and asked "does there exist a **cosmic censor** who forbids the appearance of naked singularities, clothing each one in an absolute event horizon?" This is now called the weak cosmic censorship conjecture which states in essence that all singularities of gravitational collapse are hidden within black holes. However without adding further conditions there exist counterexamples so the task in proving the conjecture is really one of finding a suitable formulation for which it is true. In 1979 Penrose introduced a second version of the conjecture, now called the strong cosmic censorship conjecture that roughly speaking says that evolving a spacetime from generic initial data does not produce a singularity visible from infinity. Again the main issue here is giving a suitable formulation for which it is true or for which there are counterexamples.

As well as his contributions to our understanding of gravitational singularities Penrose has made numerous other seminal contributions to the study of general relativity. Because of lack of space I can only talk about these in general terms. One thread running through his work is the role of null geodesics and conformal geometry and in particular how this manifests itself in understanding both the asymptotic structure and causal structure of spacetime. His first publication in this area was on the spinor approach to general relativity. This led to a number of publications with Ted Newman which introduced the NP formalism and applied this to the study of gravitational radiation. He then combined these ideas with the notion of conformal compactification to give a new description of null infinity. This enabled him to reformulate the expressions for the flux of gravitational radiation and the Bondi expression for mass-loss as well as obtain new conservation laws. The conformal description of infinity and the so-called Penrose diagrams illustrating the causal structure of the (compactified) spacetime are now a standard research tool. On a related note his papers with Kronheimer and Geroch abstracted the causal structure of spacetime in terms of an event set with a partial order and used this to construct a description in terms of ideal points with both singularities and infinity as boundary points. This was part of a programme to not just prove the existence of singularities but also to understand their nature.

In 1967 Penrose devised twistor theory which attempts to unite ideas from space-time with the principles of quantum mechanics. Twistor space is a 3-dimensional complex projective space which in a certain sense is built from the spinor representation of a null line in Minkowski space. The basic idea is to encode information about massless fields defined on Minkowski space into complex analytic objects on twistor space via the Penrose transform. Thus information about fields satisfying partial differential equations is encoded in the geometry of twistor space. The transform involves computing contour integrals of holomorphic functions (representing the free data) on regions in twistor space. In 1976 the correspondence was generalised to give a solution of (anti-)self-dual solutions Einstein's equations in terms of data on curved twistor space. However one would really like to have a single description that allows for both self-dual and anti-self-dual solutions which can then be combined to give the general solution but, despite considerable effort, limited progress has been made in achieving this. However the mathematics generated by twistor theory is very rich and has had many fruitful applications in physics. These include the definition of quasi-local mass in general relativity, the ADMH construction of Yang–Mills–Higgs monopoles and the very active area of twistor string theory which has been used to compute scattering amplitudes in gauge theories.

The limited space available prevents mentioning the full range of Penrose's research. In particular I have not mentioned quasi-periodic tiling, the Weyl curvature hypothesis and his work on cyclic cosmology nor his thoughts about the role of gravity and the reduction of states in quantum mechanics. He has also written extensively about computing and consciousness, ideas which developed from his book "The Emperor's new Mind". However I would like to end by saying something about how I first got to know about the work of Roger Penrose. It was long before I knew anything about general relativity and was not on the topic of mathematical physics at all. It came from looking at the "Ascending and Descending" staircase lithograph by M C Escher that was based on the impossible triangle of Penrose which he devised in 1958 and was published in the British Journal of Psychology. Penrose's love of geometry clearly extended beyond mathematical formulae and a distinctive feature of many of his papers and talks are the wonderful diagrams that enable him to describe four-dimensional geometry using two-dimensional drawings.

James Vickers is a Professor of Mathematics in the School of Mathematics at the University of Southampton, UK. He has been a professor at Southampton since 2001 and is part of the General Relativity research group. His recent research has involved looking at low regularity solutions of Einstein's equations and he has recently proved low regularity versions of both the Penrose and Hawking singularity theorems.