Redefining SI Base Units with Fundamental Constants

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Fig.1: Promotional illustration for the revised International System of Units.

The International System of Units (SI for short) is made up of seven base units along with derived units created through their combination. At a November 2018 meeting of the General Conference on Weights and Measures (CGPM) in Versailles, France, four of the base units—the kilogram (kg), ampere (A), Kelvin (K), and mole (mol)—respectively, were given new definitions based on the Planck constant (h), the elementary charge (e), the Boltzmann constant (k), and the Avogadro constant (N_A). As a unit of length, the meter had previously been redefined in 1983 from another fundamental constant, namely the speed of light in a vacuum (c)—a change that served as a guide for the latest unit redefinition. In this essay, I will use the process of the meter's redefinition as a unit to understand the principle behind the kilogram's redefinition and investigate the way in which the unit is realized.

"Fundamental constants" are physical quantities that remain unchanged over time and have been incorporated into the laws of physics. The speed of light in a vacuum (henceforth "speed of light") was theoretically proven to have a fixed value by James Clerk Maxwell in 1865. Einstein subsequently used its invariance as a basis for his proposal of the special theory of relativity in 1905.

Experimental physicists spent a long time using different methods to estimate the fixed value of the speed of light. A conclusive finding was finally obtained in 1972 by K. M. Evenson *et al.* from the US National Bureau of Standards (NBS). Simultaneously measuring the frequency (f) and wavelength (λ) of a methane stabilized helium-neon laser, the team worked out the speed of light by multiplying the two ($c = f \times \lambda$). The value they found for c was 299 792 456.2 $\pm$ 1.1 m/s. The "$\pm$1.1" here indicates the uncertainty of the last two digits of the average (the "6.2" at the end). The ratio of uncertainty to the average value is referred to as the "relative uncertainty," which totals $\pm 3.7\times 10^{-9}$ for the case above.

Measuring the frequency of light was no easy matter at the time. The researchers were able to reach the microwave level by using phase locking technology to divide out the frequency of light over several stages. This was a simple and accurate way to measure microwave frequency. As a standard for measuring microwave frequency, the experiment used the frequency from a cesium clock, which represented the primary standard to define the second. A second is defined according to the frequency $\Delta\nu_{Cs}$ between the two hyperfine levels of the ground state of a cesium-133 atom. In terms of a formula, it is defined by the relationship $s = Hz^{-1}$ from the value $\Delta\nu_{Cs} = 9\ 192\ 631\ 770$ Hz. The relative uncertainty (that is, the accuracy) of a cesium clock at the time was on the order of 10^{-11}. The frequency of light was sought by multiplying and adding back from the incrementally lowered frequency. As a result, the frequency of light could be obtained with a relative uncertainty of $\pm 6 \times 10^{-10}$.

The wavelength of light, in contrast, was measured with a Michelson interferometer. The standard used for measuring wavelength was the 606 nm wavelength of light from a krypton-86 atom, which defined the meter at the time. At the same time, it was known that the distribution of the 606-nm light was not symmetrical to the left and right of center, which allowed the meter to be determined with a relative uncertainty of $\pm 3.5 \times 10^{-9}$. The upshot of this was that the relative uncertainty for the

(Illustration by Kim Min-jeong).

speed of light obtained by Evenson *et al.* was determined by the uncertainty of the meter as a unit of wavelength. In other words, they had reached a point where the speed of light (which is expressed in units of meter per second [m/s]) could not be measured any more precisely. Scientists subsequently decided to fix the value of the speed of light, which was defined by CGPM in 1975 as $c = 299\ 792\ 458$ m/s. That is a fixed number, with no uncertainty. If applied to the case of the meter, it translated into $m = c \cdot 1/299\ 792\ 458 \cdot s$. This equation, in so many words, became the definition of the meter adopted by CGPM in 1983: "The metre is the length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second."

The redefinition of the meter resulted in more diverse ways of achieving one. In other words, because c (expressed in m/s) has a fixed value in the relation $\lambda = c / f$, it could be used to give the wavelength λ (expressed in m) for any light whose frequency f (expressed in Hz) had been accurately measured. (In this case, f would have to be measured against the cesium clock's frequency used to define the second. The best cesium clocks today have relative uncertainty on the order of 2×10^{-16}.) For example, a laser stabilized to the transition line of an ytterbium-171 clock (used as a standard for optical frequency) could be used to determine the accuracy of a length equivalent to the measurement accuracy for that frequency. The relative uncertainty for the laser's frequency and wavelength would equal 5×10^{-16}—an improvement of roughly 40 000 times over the iodine stabilized helium-neon laser developed in the 1980s and widely adopted as a standard for length. This is what is being sought with the latest unit redefinition: the ability to realize units with greater accuracy thanks to developments in science and technology. What makes this possible is the fact that fundamental constants do not change in value unless the very laws of physics change (in this case, c is invariant, and $c = f \times \lambda$), and the definitions of units derived from this are likewise unchanging.

For a unit to be redefined, a device must first be developed to realize the unit. That device must also allow for the highest possible level of accuracy at the corresponding term when measuring the fundamental constants' values. Systematic errors may end up being incorporated into the measurement results if only one device is used, or if measurements are conducted by only one research institution. At the very least, findings must be obtained from measurements by different institutions using differ-

Fig.2: Fundamental constants incorporated into the definition of the kilogram (kg).

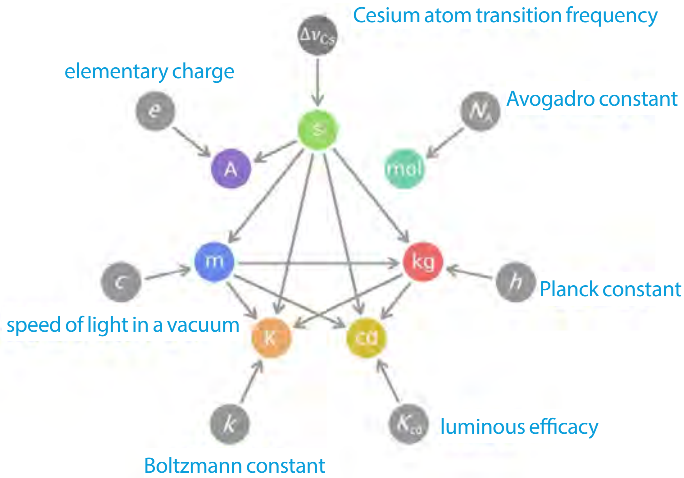

Fig.3: Relationships of the revised SI basic units: Arrows show that the corresponding fundamental constant or base unit is incorporated into the unit indicated. (e.g., A second is included in all five units apart from the mole [mol].)

ent methods (or devices). Those measurement values also need to correspond within a certain level of uncertainty. The specific requirements for redefinition were spelled out in recommendations by the Consultative Committees (CC) for the respective units under the International Committee for Weights and Measures (CIPM).

The redefinition of the kilogram was based on the Planck constant (h). The device used to realize the redefined kilogram is the Kibble balance. The idea was first proposed in 1975 by Bryan Kibble of the United Kingdom's National Physical Laboratory (NPL), with research efforts on the balance's development begun in earnest in 1990. In 2009, the balance passed into the hands of Canada's National Research Council (NRC), having failed to achieve the target uncertainty in its measurement of Planck's constant. There, a measurement finding was produced that met the conditions for redefining a unit. Another satisfactory result was also achieved with the Kibble balance by the US National Institute of Standards and Technology (NIST). An "international Avogadro project" was also carried out with the so-called XRCD experiment, which achieved a satisfactory result in its attempt to find both the Avogadro constant and the Planck constant simultaneously from a one-kilogram silicon sphere.

A special meeting of the Committee on Data for Science and Technology (CODATA) was held in 2017 for the revision of the SI, with adjusted values and relative uncertainty determined on the base of the four fundamental constant values as measured to date. For the Planck constant, the recommended value for h was 6.626070150(69) $\times$ 10^{-34} J s, with a relative uncertainty of 1.0×10^{-8}.

In 2018, CGPM announced fixed values of the fundamental constants with no uncertainty. For example, the Planck constant h was announced as "6.626 070 15 $\times$ 10^{-34} J s, where J = kg m^2 s^{-2}." In other words, h could be used as 6.626 070 15 $\times$ 10^{-34} kg m^2 s^{-1}. When applied to kilograms, the formula gives kg = h m^{-2} s / (fixed number). If the definitions of the meter and second (m = c s / 299 792 458, s = 9 192 631 770 / $\Delta\nu_{Cs}$) in place of m and s, this gives kg = (constant number) $\times$ $h\ \Delta\nu_{Cs}$ / c^2. The kilogram has ultimately been defined in terms of the Planck constant (h), the speed of light in a vacuum (c), and the transition frequency of the cesium 133 atom ($\Delta\nu_{Cs}$).

With the two-armed scale, mass is compared by placing an item to be measured on one arm and a standard counterweight on the other. In other words, the gravity operating on both arms is used to compare mass. While gravity also operates on substances measured with the Kibble balance, the comparison is based on the electromagnetic force generated on the standard arm. "Electromagnetic force" here refers to the Lorentz force ($F = BiL$) generated in a coil (length L) placed in a space with a magnetic field (B) with the flow of an electric current (i). Since it is not possible to measure the current i with a high level of accuracy, however, Ohm's law ($i = V_1 / R$) is used with the voltage measured by a Josephson quantum voltage standard and resistance (R) measured with the quantum Hall resistance standard. It is also difficult to accurately determine the BL value, which is found using through equation $BL = V_2 / v$, using

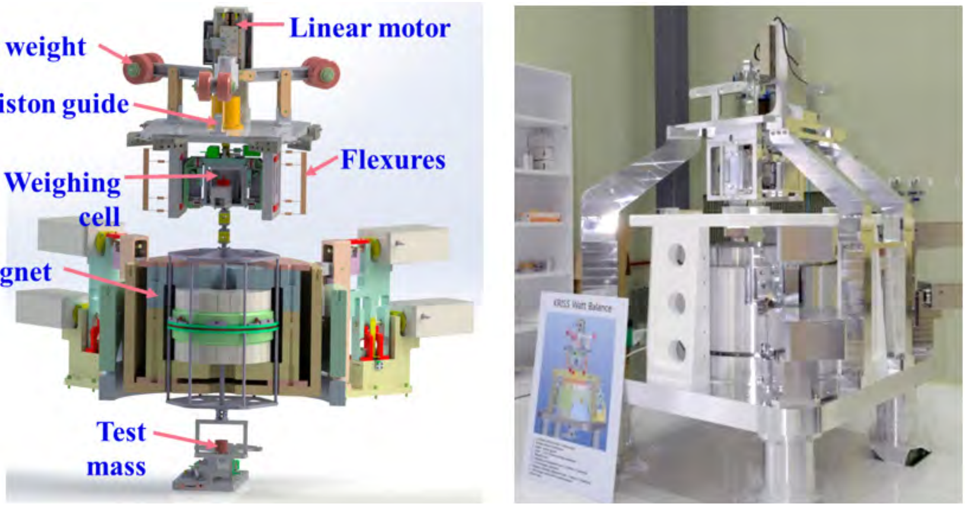

Fig.4: Diagram and photograph of a Kibble balance under development by KRISS.

the voltage (V_2) produced at the resistor ends when the standard arm (the coil here) moved at a certain velocity v. In this case, the velocity v can be accurately measured with a laser interferometer, and the voltage V_2 can be measured using a quantum voltage standard. This means that two experiments have to be conducted for the Kibble balance to produce a mass value for the substance being measured.

The International Prototype of the Kilogram (IPK) that once defined the unit has now passed into history. It came up as the first target for redefinition after it became known that the mass had changed by around 50 micrograms over a period of 100 or so years; the new definition is now based on the Planck constant. This redefinition of units will not mean any immediate changes for industries or the scientific community—the measurement science world has committed great efforts over the past decades to ensure that such changes do not happen (or, in other words, that no correction is needed). The redefined units are effective from May 20, 2019, the World Metrology Day.

Fig.5: International Prototype of the Kilogram (IPK).

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