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Recent Progress on the Non-Abelian $\nu = 5/2$ Quantum Hall State

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ABSTRACT

The fractional quantum Hall (FQH) effect at the Landau level (LL) filling factor 5/2 has attracted great attention in the physics community for over 30 years. The possibility of supporting fractionally charged quasiparticles that obey non-Abelian statistics has been the focus in recent years. In this article, progress in the exotic FQH state is reviewed, with an emphasis on the exploration of the nontrivial statistics, which may be used to realize topological quantum computation. Recent developments include the measurement of thermal Hall conductance, quasiparticle tunneling and interferometry.

INTRODUCTION

The FQH effect in two-dimensional electron systems is a remarkable demonstration of topology in strongly correlated electron systems [1]. The effect occurs when electrons are occupying a fraction of the topmost LL and the fraction normally comes with an odd denominator, required by the Pauli exclusion principle. The most common ground states are quantum liquids, which can be regarded as integer quantum Hall liquids for composite fermions, which are composed of electrons attached with an even number of vortices [2]. Therefore, the discovery of the FQH plateau at the LL filling factor $\nu = 5/2$ in 1987 [3] challenged the condensed matter physics community for many years.

Read and Moore in 1991 proposed that FQH wave functions can be constructed as conformal blocks in certain conformal field theories (CFTs) [4]. They showed that the Laughlin state at primary filling factors $\nu = 1/m$, where m is an odd integer, can be interpreted as conformal blocks of chiral boson CFTs. On the other hand, they obtained a Pfaffian wave function for $\nu = 1/2$ from the combination of the chiral Majorana (or Ising) CFT and an appropriate chiral boson CFT. Apparently, the spin-polarized Pfaffian state can be a candidate for the 5/2 state, modulo two filled LLs. However, the establishment of the relevance between the CFT-constructed wave function, which is the exact ground state of a short-range three-body interaction [5], and the experimentally observed plateau [3] took many years. Morf showed that the ground state of a microscopic system at $\nu = 5/2$ with Coulomb interaction was spin-polarized and incompressible, and had a significant overlap with the Pfaffian state [6]. Rezayi and Haldane also provided evidence that the ground state was better described as the particle-hole symmetrized Moore-Read Pfaffian state, which happened to be in proximity to a striped state [7]. The further complication of particle-hole symmetry will be discussed in greater detail along with the experimental measurement of the thermal Hall conductance. It suffices to mention here that the Moore-Read state is not particle-hole symmetric, and its particle-hole conjugate, known as the anti-Pfaffian state [8, 9], is also a strong candidate for the $\nu = 5/2$ FQH state. They have the same density and degenerate energy, but their topological properties differ, which can be distinguished in tunneling and thermal Hall conductance measurements.

The Moore-Read state can also be interpreted as a Bardeen-Cooper-Schrieffer (BCS)-paired wave function projected on to real space for composite fermions [4]. In particular, its long-distance behavior is consistent with that of the spinless p-wave weak-pairing phase [10]. In this picture, the smallest quasihole excitations, which have electric charge e/4 and a Majorana zero mode, correspond to half-flux quantum vortices in the superconducting phase. Correspondingly, the edge excitations of the paired FQH state contain a charged bosonic branch and a neutral Majorana branch [11]. For the Pfaffian state with an even number of electrons, the edge Majorana fermions obey antiperiodic boundary conditions. When charge-e/4 quasiholes are inserted into the bulk of the 5/2 state, the fermionic branch of the edge spectrum exhibits an odd-even alternation, which corresponds to switching between periodic and antiperiodic boundary conditions [12].

The alternation of the edge spectrum is an implicit numerical demonstration of the non-Abelian braiding statistics of the charge-e/4 quasiparticles [4]. More explicit considerations include the ground state degeneracy of quasiparticle states and the adiabatic transport of quasiparticles [4]. Based on concrete wave function construction, Nayak and Wilczek showed that the wave functions for 2n quasiholes at fixed positions span a 2^{n-1}-dimensional ground state manifold, in which the braiding of quasiholes generates SO(2n) rotations [13]. A rigorous proof based on a plasma analogy has established that the non-Abelian statistics of the quasiholes in the Moore-Read Pfaffian state can be obtained by the explicit analytic continuation of the CFT-constructed wave functions [14]. The adiabatic braiding of quasiholes and, hence the non-Abelian statistics, have been demonstrated numerically by Monte Carlo simulation [15, 16], by exact diagonalization [17], and by the matrix product state technique [18].

The degenerate states of non-Abelian quasiparticles are perfect for encoding information in a quantum system. Adiabatic braiding of quasiparticles leads to a unitary evolution of the quantum system, hence a quantum gate in the encoding space. The space is protected from decoherence by an excitation gap and the quantum evolution is insensitive to the deformation of the worldlines of the quasiparticles in the (2+1)d spacetime, which may be caused by local noises. This appealing scheme of encoding and manipulating quantum information is known as topological quantum computation [19], whose need for error correction is minimal. The FQH state at $\nu = 5/2$ is among the leading candidates for realizing a scheme with error protection at the hardware level. In the quantum Hall system, quantum information encoded in the final state can be read via quasiparticle interferometry.

Therefore, the most bizarre but potentially useful concept in the study of the FQH state at $\nu = 5/2$ is the non-Abelian statistics of quasiparticles. Quasiparticle tunneling, quasiparticle interference, and thermal Hall conductance measurements have yielded fruitful results for our understanding of this mysterious state. In this article, recent developments in these areas, especially experiments that challenge our earlier understanding and their theoretical analyses, will be reviewed.

QUASIPARTICLE TUNNELING

The ground state of a pure FQH state, such as a Laughlin state, exhibits uniform density in the bulk, just like Fermi liquids in metals. But the quasiparticle excitations in the FQH state carry fractional charge, which can lead to nontrivial transport signatures as they travel along the sample boundaries. According to the chiral Luttinger liquid theory, when tunneling occurs between opposite FQH edges at a quantum point contact (QPC), the tunneling conductance has a characteristic power-law dependence on temperature or bias voltage [20-23]. In the weak tunneling regime, the exponent of the power-law behavior is a universal constant that depends only on the topological properties of the corresponding FQH state, at least in the absence of edge reconstruction [24-26].

Radu et al. performed the first tunneling experiment at $\nu = 5/2$ and extracted the exponent from the temperature dependence of the zero-bias Hall resistance across the QPC [27]. Their result suggested that the 5/2 state in their sample was in the anti-Pfaffian phase [27], with statistics improved in later experiments [28]. Fig. 1 shows the resistance versus bias current curves at different temperatures. The curves can be simultaneously fitted to extract the effective charge and the interaction strength of the quasiparticles. However, measurements with other samples also found an exponent that had been predicted for the Abelian 331 wave function [29]. Yang and Feldman analyzed the tunneling data and the neutral mode measurement [30] and argued that a 5/2 liquid with Halperin's 113 topological order could be a serious candidate for explaining the experimental observations [31]. Therefore, further experiments are needed.

Very recently, Fu et al. showed that both the Abelian and non-Abelian states could be stable in the same device but under different QPC voltages, which tuned the confinement of the electron gas in the QPC [28]. Interestingly, the non-Abelian state, which had a tunneling exponent consistent with that of the anti-Pfaffian state, occurred when the QPC voltage was less negative, i.e. at weaker confinement. This is consistent with the early semi-realistic simulation that predicted that the anti-Pfaffian state emerged in the presence of a weak confining potential [32]. However, the revisit of the simulation results suggested that the Pfaffian state might evolve into an intermediate non-Abelian striped state when the confinement was weakened [33]. In any case, the experiment raised a caution that the FQH state in a QPC may be sensitive to the applied confinement potential, which deserves attention when QPCs are used in interference experiments and in potential application in topological quantum computation.

FEBRY-PEROT INTERFEROMETER

In a Hall bar with two QPCs, quasiparticles can tunnel at any of the two QPCs, and the different paths can interfere. Chamon et al. first proposed using a double point-contact interferometer to measure the fractional charge and fractional statistics of the quasiparticles in the FQH regime [34]. In fact, a Febry-Perot type interferometer can probe non-Abelian statistics [35-37] and, therefore, can be used for braiding and reading out in topological quantum computation [38]. Explicit calculations for the Pfaffian and anti-Pfaffian states at $\nu = 5/2$ showed that current oscillations occur only when the interferometer encloses an even number of non-Abelian quasiparticles [39]. Due to the smallness of the neutral mode velocity [40], the interference pattern and the even-odd effect should appear when the interferometer is sufficiently small [32, 39].

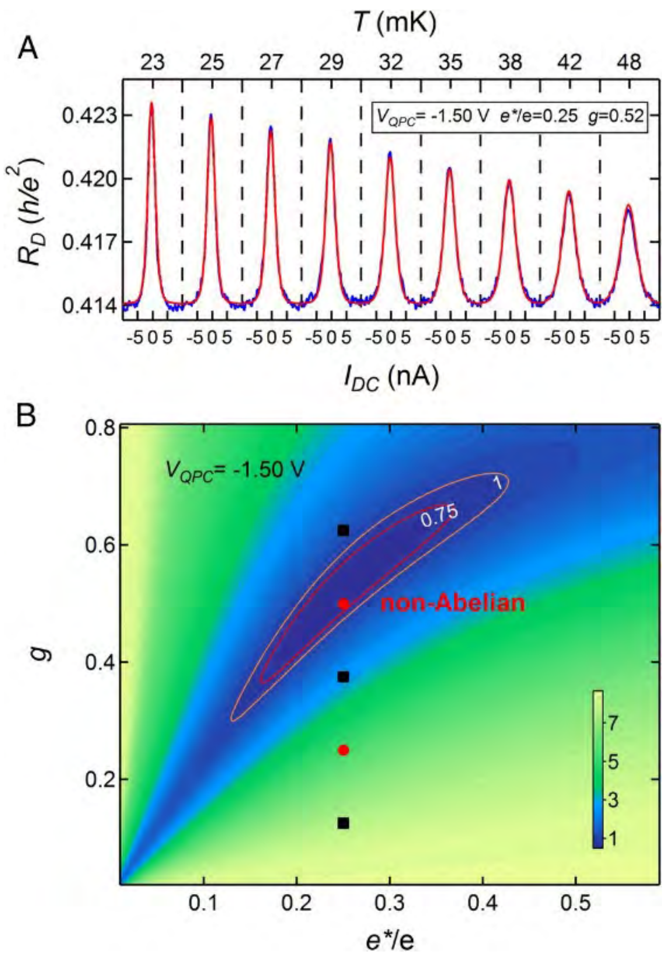


Fig. 1: Quasiparticle tunneling result for the $\nu = 5/2$ state, which is consistent with the anti-Pfaffian phase. (A) Direct current bias and temperature dependence of the diagonal resistance (blue line) and the least-square fitting curve (red line) at magnetic field B = 4.80 T and QPC voltage -1.50 V. (B) Fit residual divided by the experimental noise as a function of e*/e and g. The best fit yields e*/e = 0.25 and g = 0.52. Reprinted from Fu et al., PNAS 113, 12386 (2016).

Willett et al. first demonstrated quasiparticle interference effects in a small-area double point-contact interferometer with ultrahigh mobility and high electron density [41]. They found resistance oscillations in both magnetic-field sweeps and side-gate voltage sweeps. Furthermore, two periods of oscillation appeared in the voltage sweeps, as shown in Figs. 2(A) and 2(B). One was consistent with charge e/4 for the non-Abelian quasiparticles, while the other was consistent with charge e/2. Figs. 2(C) and 2(D) show that the e/4 oscillations were suppressed but the e/2 oscillations persisted at intermediate temperatures. In fact, numerical calculations predicted the existence of charge e/2 signals in interference [32]. The appearance of both charge e/4 and e/2 periods is the consequence of a subtle competition between the edge mode velocities [32, 39] and the tunneling amplitudes [42, 43] of the non-Abelian charge-e/4 quasiparticles and the Abelian charge-e/2 quasiparticles, both relevant to the inter-edge tunneling. The charge e/2 quasiparticles have no neutral mode, which propagates with a much smaller velocity, and, therefore, suffer less from thermal smearing and persist at relatively higher temperatures [40].

Willett et al. emphasized the alternation of e/4 and e/2 oscillations in a more careful study of the interferometer [44]. With a wide-range side-gate voltage sweep, they showed multiple aperiodic alternations in the longitudinal resistance traces. This can be attributed to the parity change of the number of non-Abelian quasiparticles inside the interference loop, whose size decreases as the voltage increases. The aperiodic alternation, whose pat-

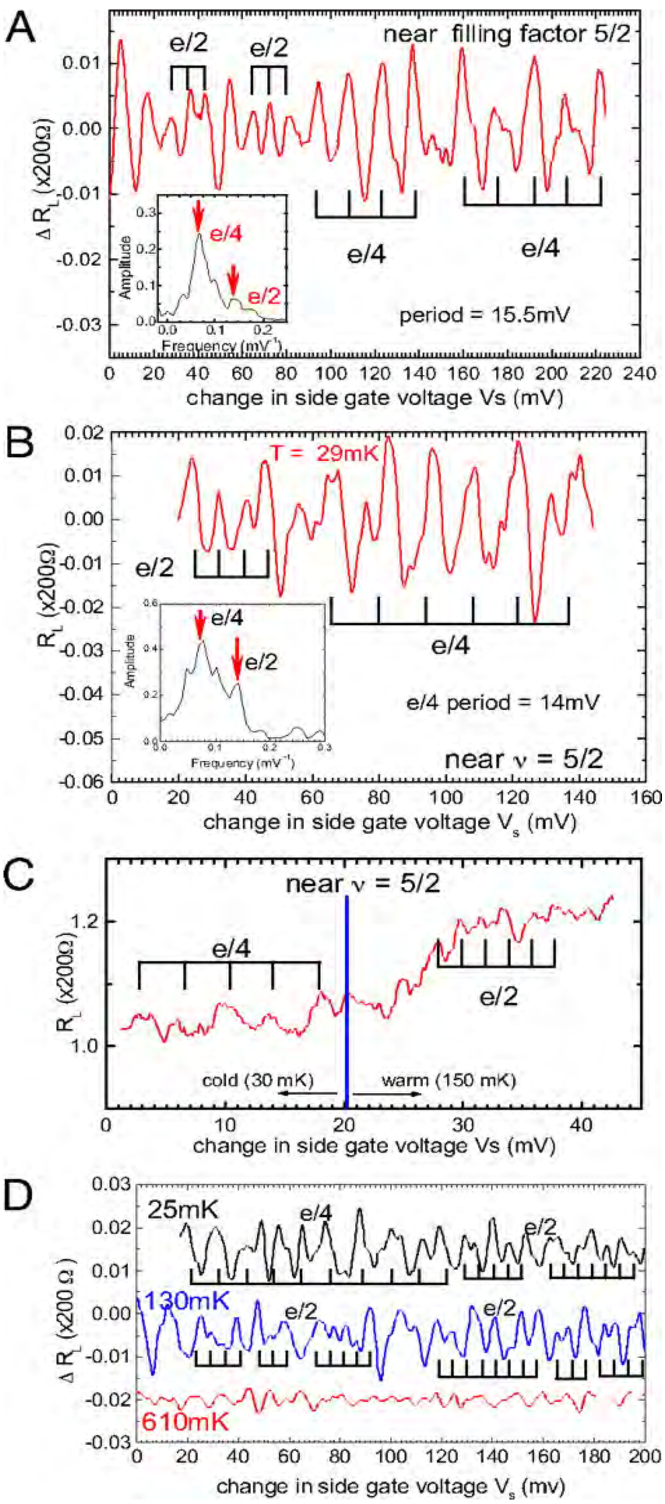


Fig. 2: Quasiparticle interference in a double quantum point contact structure in the vicinity of ν = 5/2 filling. Two different samples (A) and (B) demonstrate the presence of both e/4 and e/2 period oscillations in longitudinal resistance near filling factor 5/2 as the side-gate voltage changes at temperature T = 29 mK. The e/4 and e/2 periods are indicated as peaks in the Fourier transform in the insets. (C) Higher temperature suppresses the e/4 oscillations, leading to visible e/2 oscillations. (D) Longitudinal resistance at 25, 130, and 610 mK. Data are offset for clarity. Reprinted from Willett et al., PNAS 106, 8853 (2009).

tern depended on sample preparation, reflected the fact that the quasiparticles were trapped at random locations inside the two-dimensional electron gas. Therefore, the alternation was a manifestation of the even-odd effect with the additional charge e/2 oscillations. Remarkably, when the perpendicular magnetic field was changed by 19 G, which corresponded to the addition of a single non-Abelian quasiparticle, or the change of number parity, inside the interference loop, the e/4 and e/2 oscillations interchanged [44].

The side-gate voltage sweeps can be complemented by magnetic field sweeps. Willett et al. swept the magnetic field and found that the resistance oscillations observed near ν = 5/2 showed a period consistent with two parity change of the charge e/4 quasiparticles [45]. The oscillation period in units of flux is approximately independent of the device area [45]. Assuming constant charge inside the constant-area interference loop as the magnetic field was varied, one expects such a period for the Moore-Read Pfaffian or the anti-Pfaffian state with the even-odd effect, but not for Abelian states, such as the Halperin 331 state.

While the quasiparticle interference results are broadly consistent with theoretical predictions based on the Pfaffian or anti-Pfaffian phase, concerns have been raised regarding the role of Coulomb interaction [46]. In addition, the area of the interference loop was found to be significantly smaller than the lithographic area, indicating the edge being smooth and susceptible to edge reconstruction [24-26]. Further explorations are, therefore, needed to resolve these issues, especially for implementing a stable topological qubit in the 5/2 system.

PARTICLE-HOLE SYMMETRY

Even though evidence mounts that the 5/2 state observed in various experiments can support quasiparticles with charge e/4, there can still be multiple candidates for the state, especially when edge reconstruction is considered [47]. The most studied are the Moore-Read Pfaffian state and its particle-hole conjugate, the anti-Pfaffian state, due to their non-Abelian nature. In the presence of particle-hole symmetry the two states are degenerate. However, realistic considerations, such as LL mixing and edge confinement potential, break the symmetry.

In realistic systems, the LL spacing is not significantly larger than the Coulomb interaction. Therefore, LL mixing due to virtual excitations to other LLs induces effective three-body interaction among electrons, which breaks particle-hole symmetry. However, concluding which state would be favored energetically remains elusive in numerical calculations [48-54]. Experimentally, the LL mixing can be tuned by changing the carrier density. A recent experiment found a change of slope in the density dependence of the excitation gap for the 5/2 state, which may be caused by a topological phase transition in the ν = 5/2 FQH state [55]. However, neither the Pfaffian state nor the anti-Pfaffian state was identified in the experiment.

A second mechanism to break the particle-hole symmetry is through the single-particle edge potential that arises ubiquitously in mesoscopic samples with boundaries. Numerical calculations in the disk geometry are most suitable to study this mechanism. In this geometry, both the Pfaffian and anti-Pfaffian states can be stabilized in strong and weak confinement, respectively [32]. Interestingly, there is an additional state of stripe nature in between the two phases. Numerical calculations revealed that as the confinement potential became weaker, the fermionic edge mode became soft [56], unlike in edge reconstruction transitions of a Laughlin state [57]. This led to the critical thinking that a unconventional topological state, which consists of alternating Pfaffian and anti-Pfaffian stripes, can emerge as a competing ground state to the Pfaffian and anti-Pfaffian states [33]. Fig. 3 illustrates the evolution of the ground state of a FQH droplet at ν = 5/2 as the confinement potential is weakened. At the Pfaffian–anti-Pfaffian interface of the emergent state, multiple neutral modes propagate along a single direction, while counterpropagating charge modes are gapped, as shown in Fig. 3(b). Therefore, the striped phase, unlike those observed in high LLs [58, 59], can be a quantized Hall insulator but a bulk thermal metal.

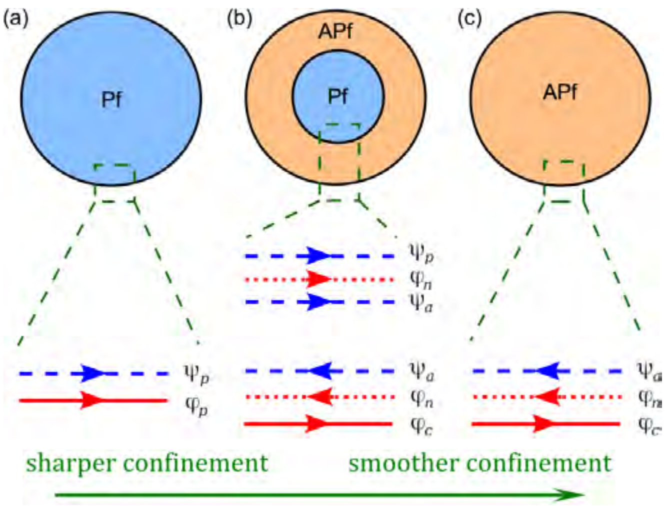


Fig. 3: Evolution of a FQH droplet at ν = 5/2. (a) The homogeneous Pfaffian (Pf) state is stable for a sharp confining potential. There is one charged bosonic mode and one neutral fermionic mode at the edge propagating in the same direction. (b) An anti-Pfaffian (APf) stripe, driven by edge reconstruction, emerges at for a smoother confining potential. At the Pf-APf interface the charged modes are gapped out, and there are only neutral modes propagating in the same direction. (c) The homogeneous APf state is stable for a smooth confining potential. In (b) and (c) we assume the APf edge modes to be strongly coupled such that there is only one charge mode, one counterpropagating neutral bosonic mode, and one counterpropagating neutral fermionic mode.

Arguably, the most unambiguous experiment to distinguish the candidates is the measurement of the thermal Hall conductance. In particular, the thermal Hall conductance is $\kappa_{xy} = \kappa\kappa_0 T$, where $\kappa_0 = (\pi^2 k_B^2/3h)$, and if the edge modes thermally equilibrate, κ is the difference between the central charges of the downstream modes and the upstream modes [60]. Hence, one expects κ = 7/2 for the Pfaffian state and 3/2 for the anti-Pfaffian state. Using a four-arm structure with a floating reservoir, Banerjee et al. demonstrated the quantization of the thermal Hall conductance at several integer and Abelian fractional filling factors [61]. Noticeable deviation occurred at ν = 2/3, possibly because of the failure of thermal equi-

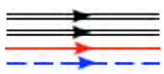
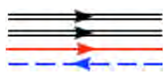
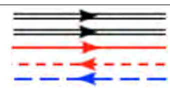
FQH States	Pfaffian	K = 8	PH-Pfaffian	113	Anti-Pfaffian
Statistics	Non-Abelian	Abelian	Non-Abelian	Abelian	Non-Abelian
Edge modes					
κ	7/2	3	5/2	2	3/2

Table I. Summary of the quasiparticle statistics, edge modes, and thermal Hall conductance coefficient of the leading candidates for the FQH state at ν = 5/2 that emerge in the disordered network model of Pfaffian and anti-Pfaffian puddles. The black double lines, red solid lines, red dashed line, and blue dashed lines are charged integer modes, charged fractional modes, neutral bosonic modes, and neutral Majorana modes, respectively. Note that in depicting the edge modes we assume full equilibration among them.

librium between counterpropagating edge modes. In a later experiment, Banerjee et al. found that the thermal Hall conductance of the 5/2 state was compatible with $\kappa = 2.5$, in sharp contrast to the theoretical expectation [62]. The half integer value is a strong indicator that the 5/2 state has a neutral Majorana edge mode with central charge 1/2. Therefore, the observation is consistent with the belief that the 5/2 state supports non-Abelian quasiparticles, but it rules out both the Pfaffian and the anti-Pfaffian states as the candidate. One possible candidate for the experimental observation is a Pfaffian-like construction that respects particle-hole symmetry, dubbed the PH-Pfaffian state [9, 63].

While the PH-Pfaffian has been argued to be stabilized by, counter-intuitively, particle-hole symmetry breaking terms [64], so far there is no strong evidence for the stabilization of the PH-Pfaffian wave function as a topological phase in numerical calculations. An alternative picture is that a FQH state at 5/2 filling with $\kappa = 5/2$ can arise from an inhomogeneous Pfaffian and anti-Pfaffian mixture [65, 66]. As already argued in the anisotropic case, such a state is particle-hole symmetric on average, but not in the local sense [33]. Mross et al. [65] and Wang et al. [66] considered alternating mesoscopic Pfaffian and anti-Pfaffian puddles as a result of quenched disorder, on whose network-like boundaries neutral Majorana fermions propagate. The problem falls into the Anderson localization of the Altland-Zirnbauer symmetry Class D [67, 68]. Mross et al. argued that a series of topologically different 5/2 phases with different κ could arise at moderate disorder, among which one is adiabatically connected to the PH-Pfaffian state with $\kappa = 5/2$ and can be stabilized at sufficiently long distances [65]. Table I depicts the edge modes and lists the corresponding κ for the relevant candidates. On the other hand, Wang et al. showed that the $\kappa = 5/2$ phase was stable only in a very restrictive parameter space [66]. For very weak disorder, direct transition between the Pfaffian and the anti-Pfaffian states can occur, while an intermediate state can appear for sufficiently strong disorder. Predominantly, the intermediate phase is a thermal metal with continuously varying κ [66].

There is one more possibility. Simon argued that the observed thermal Hall conductance could also be the result of an anti-Pfaffian ground state, whose edge states are not fully thermally equilibrated [69]. This interpretation is backed by strong numerical support, including the routinely observed anti-Pfaffian ground states and the sharp difference in the velocities of the neutral Majorana mode and the charged bosonic mode [40]. Such a lack of equilibrium can also be used to explain the anomalous κ for the $\nu = 2/3$ state in the earlier experiment [61]. However, Feldman counter argued that the interpretation is inconsistent with experimental data and the sample structure [70], but the subtleties of the edge-mode equilibrium deserve further study [71].

SUMMARY AND OUTLOOK

In summary, the FQH state at LL filling $\nu = 5/2$ remains one of the most mysterious and exciting systems in condensed matter physics. Multiple candidates compete, as interaction, confinement, and disorder are fine-tuned. Quasiparticle tunneling, quasiparticle interference, and thermal Hall conductance measurements have provided evidence that the state can support fractional charge excitations that obey non-Abelian statistics. These exotic quasiparticles can encode quantum information in a nonlocal fashion, which can be robust against local noise perturbations.

Recent developments in relation to the 5/2 state call for a more thorough study of the effect of quenched disorder on the particle-hole symmetry of the state. Experimentally, one wishes to improve the engineering of quantum point contacts with well-defined boundaries. This will facilitate the fabrication of better quantum Hall interferometers, which are of great importance for the implementation of topological quantum computation with non-Abelian fractional quantum Hall liquids.

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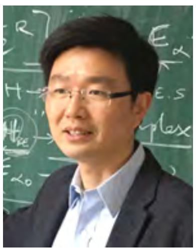

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